Atmospheric Lidar

3. Lidar models and SNR

Lidar Tutorials

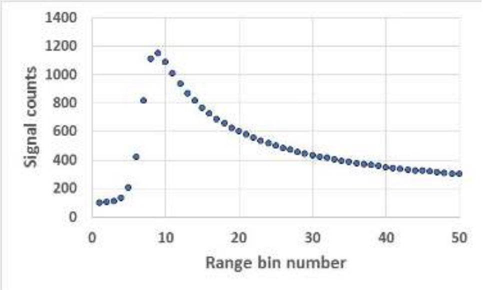
21st Coherent Laser Radar Conference

Lidar system models

- For the development of any useful sensor system, a mathematical model with inputs and outputs is required.
- The model has several uses:
  - Understanding measurements
  - Engineering new systems
  - Developing data analysis algorithms
  - Developing new techniques
- We will model the most basic type of lidar, elastic backscatter.
- The outputs of the lidar models are used to find the best possible Signal-to-Noise Ratio (SNR) of a measurement.

The lidar equation

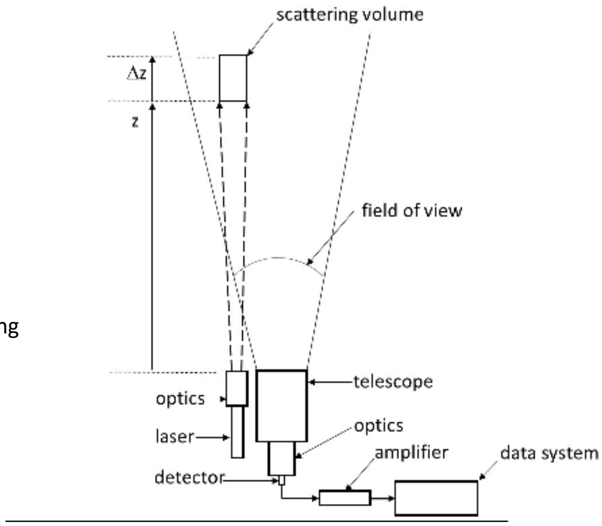
- The lidar equation is used to model lidar signals as functions of range.
- The signals are always recorded in discrete range increments known as *range bins*.
- We will model the signal as the number of laser photons received in each range bin.



G.G. Gimmestad graphic

The lidar link budget

- The lidar equation is a time-dependent *link budget*, from the laser to the detector.
- It predicts photons per range bin (or power on the detector) as a function of range z.
- We will trace the light from the laser
  - through the transmitter optics,
  - transitioning into the receiver field of view,
  - up through the atmosphere to the scattering volume,
  - through the scattering process,
  - back down through the atmosphere,
  - through the telescope and receiver optics, and onto the detector.



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Number of photons in a laser pulse

$$E_{photon} = h\nu = \frac{hc}{\lambda} \approx \frac{2x10^{-25}}{\lambda}(J/photon)$$

$$N_0 = E_{pulse} / E_{photon} \approx E_{pulse} \times 5 \times 10^{24}\ x\ \lambda(m)\ \ photons$$

Example:

λ = 0.532 µm = 0.532 x 10^{-6} m

E_{pulse} = 1 J

N_0 = 2.66 × 10^{18} photons

- N_0 is the number of photons in each laser pulse.
- The energy per photon is very small, so the number of photons per joule is very large.
- N_0 increases linearly with λ.

Losing photons in the optics

$$k_T = \prod_i T_i \prod_j R_j$$

where

$T_i = transmission\ of\ component\ i$

$R_j = reflectivity\ of\ component\ j$

Example:

Two mirror surfaces at 88% R each

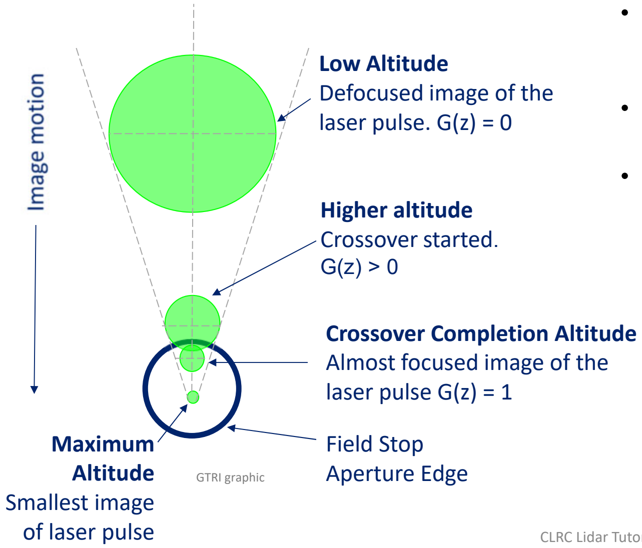
Four uncoated lens/window surfaces at 96% T each

k_T = (0.88^2 *0.96^4)*100% = 66%

One third of the laser photons were lost in the transmitter optics.

- The parameter k_T represents all losses in the transmitter optical system due to
  - Mirror reflectances <1
  - Transmittances of optical elements <1
- Losses add up *quickly*
- It is important to use a minimum number of optical components and to use anti-reflection (AR) coatings.

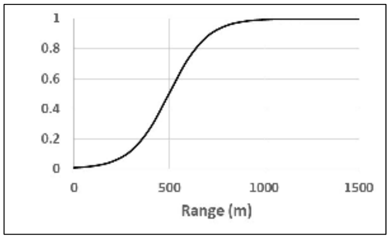
Transitioning



GTRI graphic

- A lidar receiver is an imaging system.
- At short ranges, the image of the scattering volume in the plane of the receiver's field stop is blurred by defocus and shifted to one side.
- As the pulse propagates upward, the image shrinks and moves toward the field stop center.
- The function G(z) describes the receiver sensitivity, transitioning from zero to unity as range increases.

Example of Geometrical Function G(z)



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Up through the atmosphere (and back down)

The pulse loses photons on the way up, due to extinction.

$$\sigma = \sigma_{aer} + \sigma_{mol\ scat} + \sigma_{mol\ abs}$$

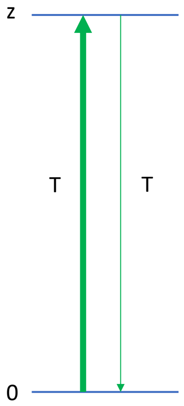
Recalling Beer's law, the transmittance from 0 to z is

$$T(z) = \exp\left[-\int_0^z \sigma(z')dz'\right] \qquad .$$

The same extinction occurs on the way back down, so

$$T^2(z) = \exp\left[-2\int_0^z \sigma(z')dz'\right]$$

is the transmittance on the two-way path.



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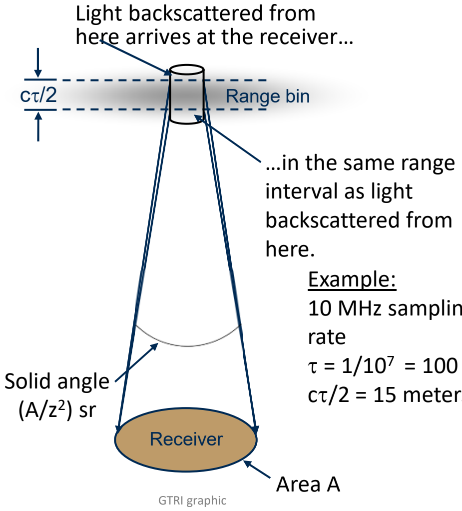
The backscattering process

$$\beta = \beta_{aer} + \beta_{mol}\ (1/m\cdot sr)$$

- The unit m (meters) refers to the range bin length. $\left(\frac{c\tau}{2}\right)$
- The unit sr refers to the solid angle subtended by the receiver at range z. $\left(\frac{A}{z^2}\right)sr$
- The scattering process is described by the product, which is the fraction of the photons that made it to the volume that were scattered back at the receiver, per steradian. $\beta\left(\frac{c\tau}{2}\right)\left(\frac{A}{z^2}\right)$



Light backscattered from here arrives at the receiver…

…in the same range interval as light backscattered from here.

Solid angle (A/z^2) sr

Area A

GTRI graphic

Example:

10 MHz sampling rate

$\tau = 1/10^7 = 100$ ns

$c\tau/2 = 15$ meters

Through the receiver and onto the detector

$$k_R = \prod_i T_i \prod_j R_j$$

where

T_i = transmission of component i

R_j = reflectivity of component j

Example:

Two mirror surfaces at 88% R each

Eight AR coated lens/window surfaces at 99% T each

One narrowband filter at 45% T

$k_R = (0.88^2 * 0.99^8 * 0.45) * 100\% = 32.2\%$

Two thirds of the photons that made it back to the receiver aperture were lost! And our *total* loss is 78%.

- The parameter k_R represents all losses in the receiver optical system due to
  - Mirror reflectances <1
  - Transmittances of optical elements <1
- Losses add up *quickly*
- It is not unusual for k_R to be as small as 0.1 due to the cumulative effect of many optical components.

The Elastic Backscatter Lidar Equation

$$N_S(z) = N_0 k_T k_R \, G(z) \left(\frac{A}{z^2}\right)(c\tau/2)\,\beta(z)\exp\left[-2\int_0^z \sigma(z')dz'\right]$$

$N_S(z) =$ Number of signal photons from range z

$N_0 =$ Number of photons per laser pulse

$k_T =$ Transmitter optical efficiency

$k_R =$ Receiver optical efficiency

$G(z) =$ Geometric crossover factor

$\left(\frac{A}{z^2}\right) =$ Receiver solid angle (sr)

$\left(\frac{c\tau}{2}\right) =$ Range bin length (m)

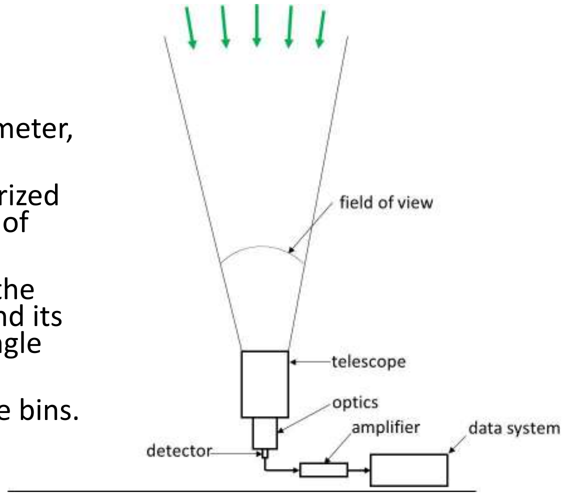
$\beta(z) =$ Atmos. backscatter coefficient ($m^{-1}sr^{-1}$)

$\exp\left[-2\int_0^z \sigma(z')dz'\right] =$ Two-way path transmittance

$\sigma(z) =$ Atmospheric extinction coefficient (m^{-1})

Background light

- Sky photons also get to the detector!
- The lidar receiver is effectively a radiometer, often pointed at the sky.
- Optical power from the sky is characterized by the *spectral sky radiance* L_λ in units of W/m^2-µm-sr.
- To find received power, multiply L_λ by the area of the radiometer receiver (m^2) and its spectral bandpass (µm) and its solid angle field of view (sr).
- The background is the same in all range bins.



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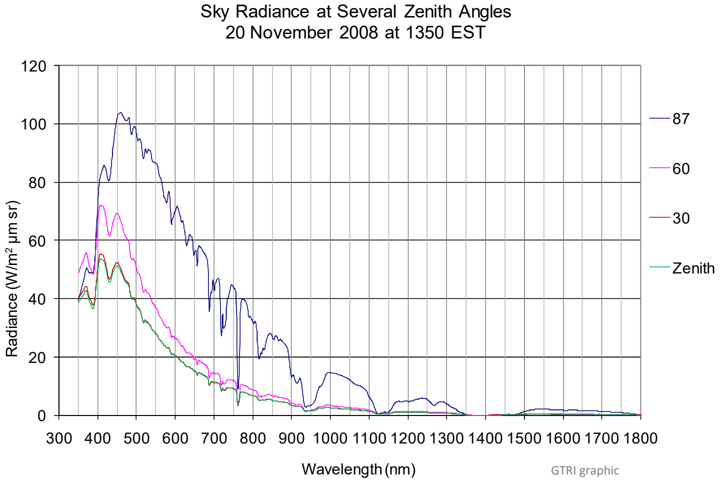
Sky radiance

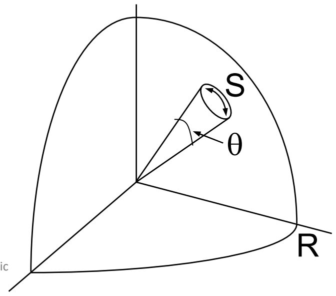
- Sky radiance peaks in the UV-Visible region, where many types of lidar must operate.
- As rules of thumb, the maximum zenith radiance is 100, and the night sky is darker by a factor of 10^{-6}.



GTRI graphic

Plane and solid angles

How to find solid angle (steradians) from plane angle (radians):



GTRI graphic

Plane angle $\theta = S/R$

Solid angle $\Omega = A/R^2$

But $A = (\pi/4)\, S^2 = (\pi/4)\, \theta^2 R^2$

So $\Omega = (\pi/4)\, \theta^2$

(for small θ)

The LIDAR receiver FOV is usually specified as plane angle

The sky background model

$$N_B = k_R L_\lambda A \Omega_{FOV} B(\tau\lambda / hc)$$

where

$N_B =$ the number of background photons per range bin

$k_R =$ receiver efficiency

$L_\lambda =$ background spectral radiance (W/m2-μm-sr)

$A =$ receiver area (m^2)

$\Omega_{FOV} =$ receiver FOV (sr)

$B =$ receiver optical bandpass (μm)

$(\tau\lambda / hc) =$ conversion from power (W) to photons/bin

Lidar SNR

- Multiply laser and background photon numbers by the detector's quantum efficiency η to convert photons to photoelectrons.
- Poisson statistics tell us that the standard deviation of the laser photoelectron count is just the square root of the total count.
- The *statistical limit* of lidar SNR is the signal (mean number of lidar photoelectrons) divided by the noise (standard deviation of all photoelectrons):

$$SNR = \frac{n_S}{\sqrt{n_S + n_B + n_D}}$$

where n_D is the number of dark counts/bin from the detector.

A lidar SNR nomogram

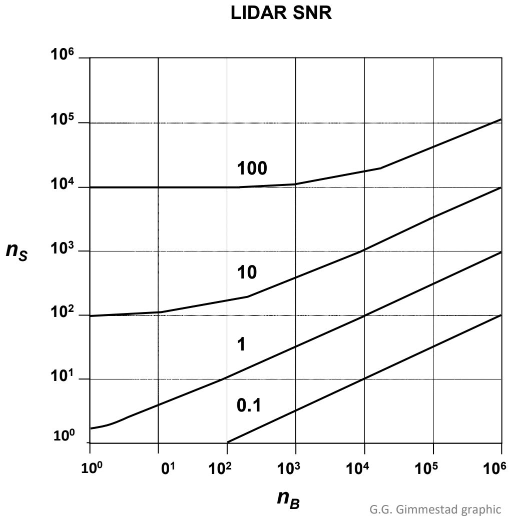
- The SNR has two limiting cases:

Signal-limited case: $n_S > n_B$

$$SNR \to \sqrt{n_S}$$

Background-limited case: $n_B > n_S$

$$SNR \to n_S / \sqrt{n_B}$$



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[n_D is assumed to be approximately 0]

Summary

- The number of laser photons per range bin increases with all instrumental parameters in the lidar equation (good).
- The number of background photons received also increases with k_{rec} and A (bad).
- The background can only be usefully minimized by
  - Decreasing the optical bandwidth (linear decrease)
  - Decreasing the receiver FOV angle (quadratic decrease)
- Many high-altitude lidars are only operated at night.